$$T(t,r) = \frac{B}{4\pi k} p^i \sum_{n=0}^{\infty} \frac{1}{n+1+i}$$

$$\times \left(\frac{p}{t}\right)^{n+1} \frac{M_n}{\eta} \exp(-\eta^2/2), \quad t > p \qquad (13b)$$

$$i > -1$$

where

$$M_n = \sum_{j=0}^{n} \frac{(-1)^j n!}{(n-j)! j!^2} \frac{\eta^{2j+1}}{2^j}.$$

It can be shown that equation (13a) converges for $t, r > 0$. The convergence of (13b) can easily be shown by comparison with the geometric series.

The temperature distribution may also be evaluated using the integral expression of equation (11). Thus

$$T = \frac{B}{4\pi k} \int_{0}^{t} \frac{t'^i \exp\left[-\rho c r^2/4k(t-t')\right]}{t-t'} dt', \quad t \leqslant p \qquad (14a)$$

$$i > -1$$

$$T = \frac{B}{4\pi k} \int_{0}^{p} \frac{t'^i \exp\left[-\rho c r^2/4k(t-t')\right]}{t-t'} dt', \quad t > p \qquad (14b)$$

$$i > -1.$$

For $p/t \geqslant 1$, the given solutions, equations (13a) and (14a) are equivalent to the classical exponential integral solution for a continuous line source. For $p/t < 1$, the solution is that of a continuous source which then decays when the source is shut off.

For the case where $i = 0$ (constant heat generation), the long and short time expansions of equation (14a) are given on p. 262 of [1]. Equations (14a) and (14b) can also be integrated by a term by term integration of the series expansion of the integrand. The resulting expression for equation (14a), for $t \leqslant p$, is found to be easier to use than equation (13a) because the former converges faster. For the same reason, equation (13b) for $t > p$, is more convenient to use than equation (14b). Furthermore, the involved numerical integration for various values of $p$ as given by equation (14b) is avoided by using the more accessible parametric representation of equation (13b). The results of the integration of the latter at various values of $p/t$ for the important case $i = 0$, is shown in Figs. 1 and 2.

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# POOL BOILING IN DILUTE NON-AQUEOUS POLYMER SOLUTIONS

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## NOMENCLATURE

$A$,   area of flat plate boiling surface, $[\text{ft}^2]$;

$\overline{M}_v$, $M$,   average polymer molecular weight, measured by a viscometric method; and molecular weight in general;

ppm,   solute concentration, in parts per million by weight;

$Q$,   rate of boiling heat transfer $[\text{Btu/h}]$;

$\Delta T$,   $T_{\text{plate}} - 177°$, driving force for boiling cyclohexane at 1 atm $[°\text{F}]$.

## INTRODUCTION

THE UNIQUE character of heat, mass and momentum transport in dilute polymer solutions undergoing non-simple flows has attracted considerable interest in recent years [1, 2]. Studies in this laboratory [3] have shown that trace amounts (60–500 ppm) of certain polymers dissolved in distilled water result in a significant increase of nucleate boiling heat flux over that for water alone. The latter work, however, was restricted in scope, in that (a) the steam-heated apparatus could not achieve $\Delta T$'s sufficiently high to reach peak heat fluxes in water and (b) the phenomenon was not demonstrated for polymers in non-aqueous liquids. These two points are examined here.

## EXPERIMENT

Cyclohexane solutions were prepared from two grades of polyisobutylene, Vistanex L80 and L100 (Enjay chemical Co., Linden, N. J.) having molecular weight [4, 5] $\bar{M}_v = 7 \cdot 2 \times 10^5$ and $1 \cdot 4 \times 10^6$, respectively. This polymer–solvent combination was selected because it is known to exhibit anomalously low friction factors in turbulent pipe flow [5] and, in other ways, is apparently a typical and common organic system. Solute concentration was varied from 51 to 1036 ppm for L80, and from 22 to 5170 ppm for L100.

The horizontal-plate pool boiler used here has been described previously [3]. It consisted of an 8 in. × 8 in. × 12 in. stainless steel box, with a circular chromium-plated copper disk set into the Bakelite bottom and heated from below by steam. Some minor modifications were made in the condensate return line to prevent recycled solvent from boiling prior to re-entry to the pool. Techniques and reproducibility of data were also similar to those reported before. Heat flux $Q/A$ was determined by measurements of the steady-state condensate rate, and the temperature driving force $\Delta T$ was determined with thermocouples embedded in the copper plate. For each liquid, viscosity was measured at 25°C both before and after boiling, with a Ubbelohde viscometer. Surface tension was not measured, inasmuch as it has already been found [3] to be only a secondary variable and not responsible for the phenomenon in question.

## RESULTS AND DISCUSSION

Boiling curves are presented in Figs. 1–3. Each liquid system manifested a nucleate boiling region (with increasing $Q/A$) at low $\Delta T$, followed by either a peak or a broad plateau which extended far into the $\Delta T$-regime wherein film boiling predominates for solvent alone. This latter region was not attainable in the earlier study [3].

Sufficiently dilute solutions are seen to be more effective heat transfer media than solvent alone, where nucleate boiling at low $\Delta T$ is involved. This corroborates the behavior of aqueous systems [3]. However, there is clearly an effect of diminishing returns as polymer concentration is extensively increased. For the L100 (Fig. 2) there exists an optimum concentration, probably near 54 ppm. For the L80 (Fig. 3), no such optimum is apparent but must obviously exist.

However, the existence of enhanced heat transfer at low $\Delta T$ appears not to lead to an increase in peak heat flux. In such cases, the peak for the solutions is comparable in
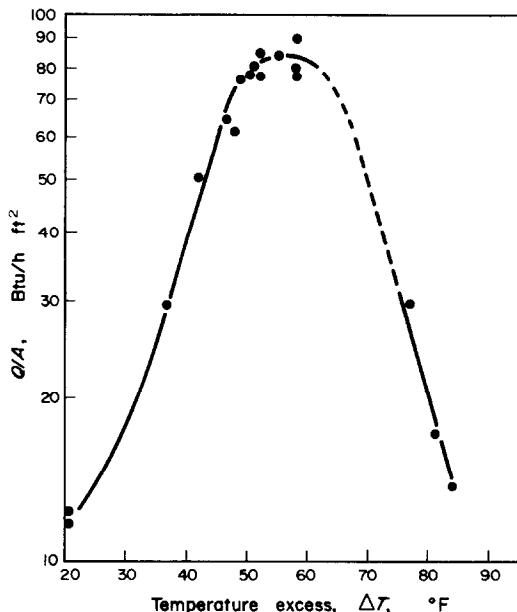


FIG. 1. Boiling heat transfer for a commercial grade of cyclohexane.

magnitude to that of the solvent. Whether this is general is unclear; data for some of the aqueous systems reported earlier [3] suggested that the peak fluxes might be enhanced. When concentrations become sufficiently high, of course, the peaks—and the whole curves—are substantially reduced. This is believed to be due to enhanced viscosity, which is well known to be detrimental to boiling heat flux [6].

An interesting alteration of the boiling curve was encountered at higher $\Delta T$, above the solvent peak flux, where film boiling customarily occurs. Whereas Fig. 1 for solvent shows the sharp peak and subsequent flux decrease as expected, the curves in Figs. 2 and 3 show very broad peaks which become plateaux persisting to the maximum $\Delta T$ available. The detailed nature of the boiling in this region is not fully characterized or understood. One can speculate that the polymer molecules encourage continued bubble nucleation at high $\Delta T$ because of their marginal solubility and possible tendency to act as nucleation sites. This seems plausible because, indeed, bubbling was clearly visible in this region. Thus the polymer presence apparently extends the effective nucleate boiling regime to higher $\Delta T$. In none of the polymer solutions was film boiling observed. In the more viscous systems the heat flux reduction was accompanied by very slow evolution of tiny bubbles and almost no bulk turbulence, but surface nucleation was still occurring.

For the aqueous systems discussed earlier [3], it was found that the influence of polymer molecular weight and concentration on the low-$\Delta T$ nucleate boiling curve could be correlated simply in terms of the solution viscosity. This concept is tested here (Figs. 4 and 5) for both the low-$\Delta T$ and the high-$\Delta T$ (post-peak) regions in an organic system.
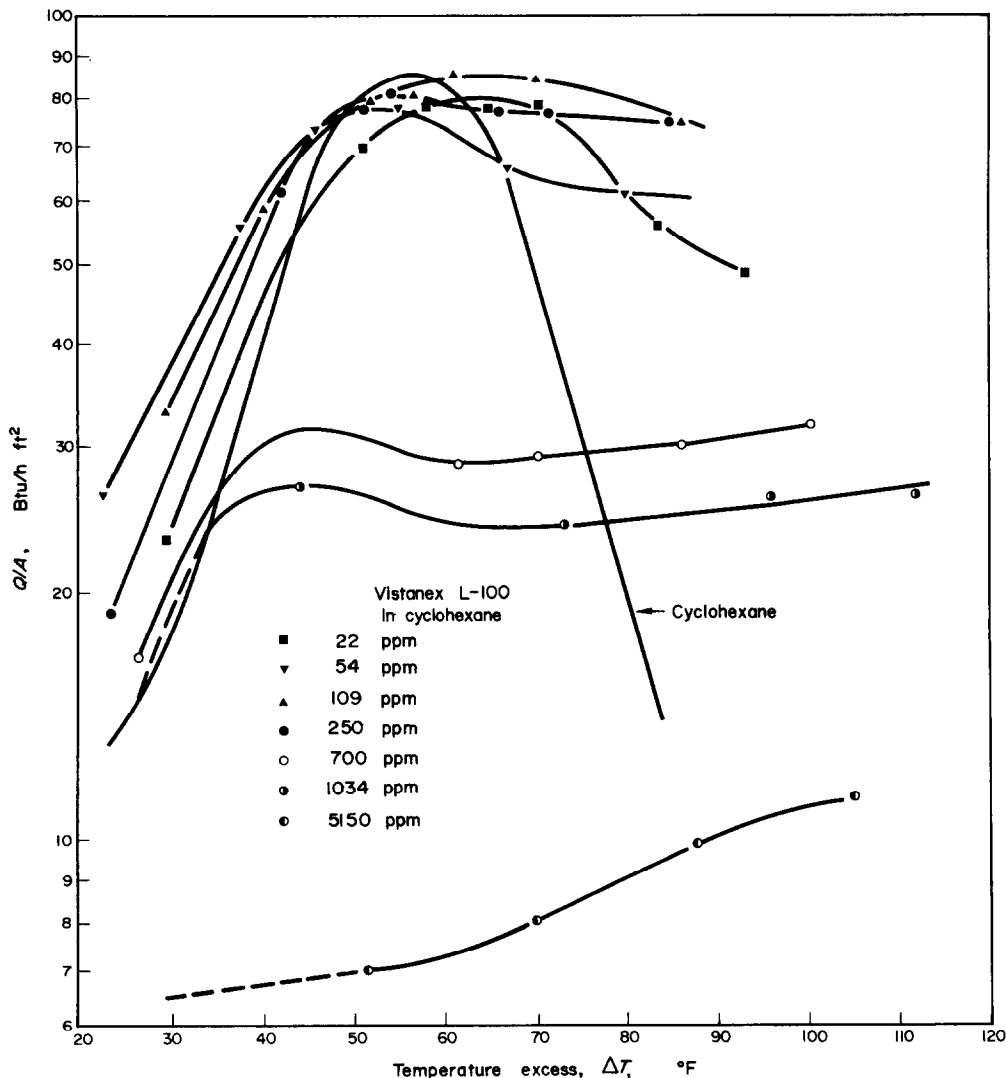
FIG. 2. Boiling heat transfer for dilute solutions of polyiso-
butylene ($\overline{M}_v = 1\cdot4 \times 10^6$) in cyclohexane.

Again, to a very good approximation, this scheme appears to correlate the observations.

Figures 4 and 5 demonstrate also that this enhancement phenomenon has the following properties:

(a) A pronounced maximum exists, indicating that such behavior can be optimized easily in terms of a viscosity parameter alone;

(b) The maximum occurs at nearly the same viscosity values for all $\Delta T$'s, about $1\cdot2$ cs for the polyisobutylene–cyclohexane system.

(c) The maximum may reflect improvements up to several hundreds of per cent, and is especially dramatic in the $\Delta T$ region usually associated with film boiling in the solvent.

Results qualitatively similar to these have been obtained by Westwater and coworkers [7, 8], who used a variety of low-M solutes in organic solvents. They also concluded that surface tension was not relevant in explaining the enhancement effect, and postulated explanations based on increased nucleation [7] and local viscosity increase near bubble surfaces to retard coalescence [8]. That the latter concept is a plausible contributing mechanism is supported by the present results, in that viscosity is a generally successful
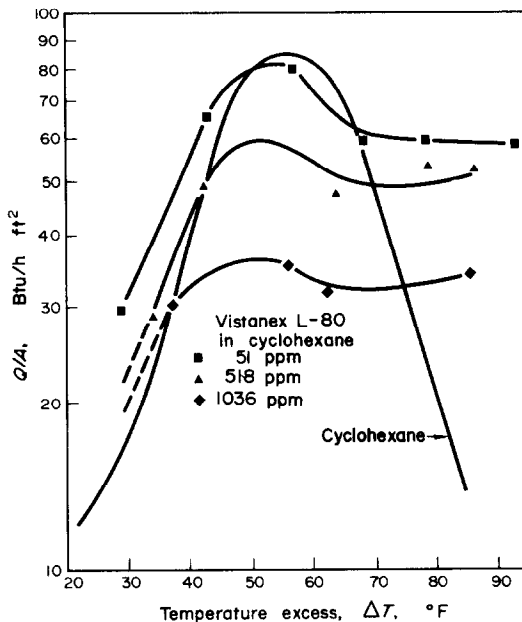
FIG. 3. Boiling heat transfer for dilute solutions of polyiso-
butylene ($\overline{M}_v = 7\cdot2 \times 10^5$) in cyclohexane.
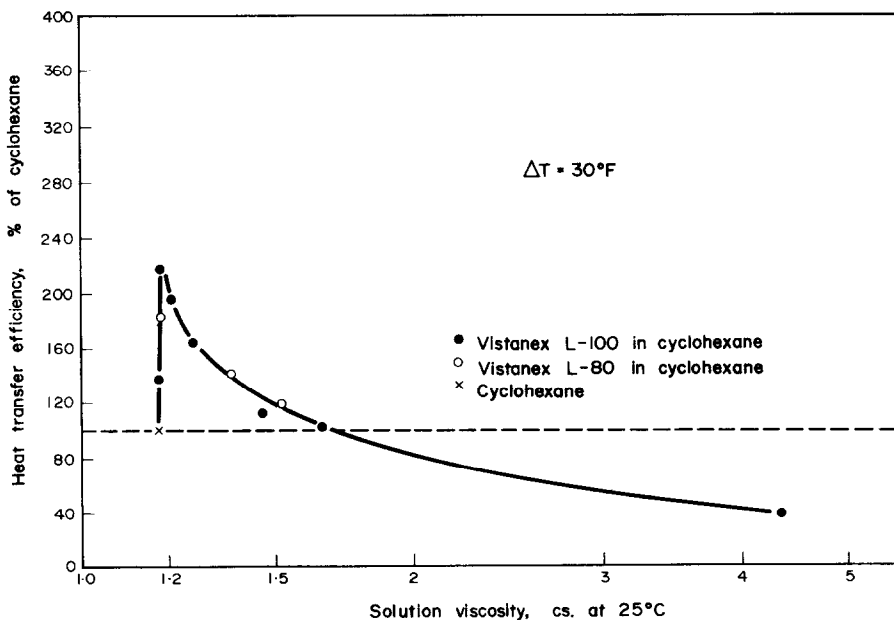


FIG. 4. Heat transfer effectiveness of the polymer solutions,
relative to cyclohexane alone, correlated with viscosity.
Points are taken from the smoothed curves of Figs. 1–3 at
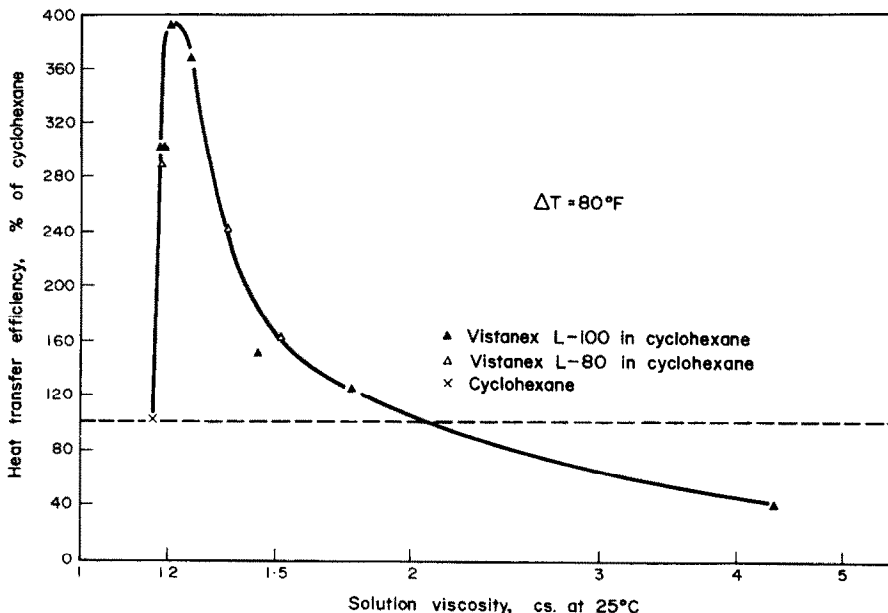$\Delta T = 30°$F, and therefore represent nucleate boiling.

FIG. 5. Heat transfer effectiveness of the polymer solutions, relative to cyclohexane alone, correlated with viscosity. Points are taken from the smoothed curves of Figs. 1–3 at $\Delta T = 80°F$, and therefore represent a comparison with film boiling in the solvent.

correlating parameter. It is interesting to note, however, that the low-M systems do not maintain such high plateau levels of flux beyond the peaks, do not in general suppress film boiling, and require much higher concentrations for comparable effects than do the polymer systems reported here.

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